Math Help Sheet: Adding and Subtracting Mixed Numbers

We begin with a quick review of adding and subtracting proper fractions.

To add or subtract two fractions it is necessary that they first be expressed as equivalent fractions because they must have a common denominator, preferably the least common denominator (LCD).

Examples: $\frac{1}{2}+\frac{1}{3}=\frac{1\times3}{2\times3}+\frac{1\times2}{3\times2}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$ and $\frac{1}{2}-\frac{1}{3}=\frac{1\times3}{2\times3}-\frac{1\times2}{3\times2}=\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$

Results should be expressed in simplest form. A fraction is reduced to lowest terms, or simplified, when its numerator and denominator have no common factors and an improper fraction is converted into a mixed number.

Working with mixed numbers

Consider expressing $\frac{17}{3}$ as a mixed number. $17 \div 3 = 5$ with a remainder of 2, so $\frac{17}{3}=5+\frac{2}{3}=5\frac{2}{3}$. Another approach is as follows: $\frac{17}{3}=\frac{15}{3}+\frac{2}{3}=5+\frac{2}{3}=5\frac{2}{3}$.

Now consider expressing $5\frac{2}{3}$ as an equivalent improper fraction. We convert the whole part into a fraction then add it to the fractional part as follows: $5=\frac{5}{1}=\frac{5\times3}{1\times3}=\frac{15}{3}$, so $5\frac{2}{3}=\frac{15}{3}+\frac{2}{3}=\frac{17}{3}$

It is important that you master conversions between fractions and mixed numbers.

Please convert the following fractions into mixed numbers: **A)** $\frac{17}{5}=?$ **B)** $\frac{21}{4}=?$ **C)** $\frac{19}{8}=?$

Please convert the following mixed numbers into fractions: **D)** $2\frac{2}{3}=?$ **E)** $3\frac{1}{4}=?$ **F)** $5\frac{5}{6}=?$

ANSWERS: A) $3\frac{2}{5}$ B) $5\frac{1}{4}$ C) $2\frac{3}{8}$ D) $\frac{8}{3}$ E) $\frac{13}{4}$ F) $\frac{35}{6}$

Method 1: For addition or subtraction with mixed numbers we do the following:
Step 1 - Convert the fractions into improper form.
Step 2 - Determine the lowest common denominator (LCD).
Step 3 - Express each fraction as an equivalent fraction having the LCD as its denominator.
Step 4 - Carry out the operation (add or subtract the numerators, place the result over the denominator).
Step 5 - Simplify the result - Convert an improper fraction into a mixed number, if necessary.
- Reduce the fractional part to lowest terms, if necessary.

Example 1: $5\frac{2}{3} + 2\frac{1}{2} = \frac{17}{3} + \frac{5}{2} = \frac{17\times2}{3\times2} + \frac{5\times3}{2\times3} = \frac{34}{6} + \frac{15}{6} = \frac{49}{6} = \frac{48+1}{6} = 8\frac{1}{6}$

Example 2: $5\frac{5}{6} - 2\frac{1}{2} = \frac{35}{6} - \frac{5}{2} = \frac{35}{6} - \frac{5\times3}{2\times3} = \frac{35}{6} - \frac{15}{6} = \frac{20}{6} = \frac{18+2}{6} = 3\frac{2}{6} = 3\frac{1}{3}$

Method 2: For addition with mixed numbers we do the following:
Step 1 - Rearrange the addition to add the whole numbers and the fraction parts separately.
Step 2 - Add the whole numbers.
Step 3 - Add the fractions as usual.
- Simplify the result - Convert an improper fraction into a mixed number, if necessary.
- Reduce the fractional part to lowest terms, if necessary.
Step 4 – Add the results from steps 2 and 3 to form your answer.

Example: $5\frac{2}{3} + 2\frac{1}{2} = (5+2) + (\frac{2}{3}+\frac{1}{2}) = 7 + (\frac{4}{6}+\frac{3}{6}) = 7 + \frac{7}{6} = 7 + 1\frac{1}{6} = 8\frac{1}{6}$

Method 2: For subtraction with mixed numbers we do the following:
Step 1 - If the value being subtracted has the larger fractional part then we borrow (see example 2).
Step 2 - Rearrange the subtraction to subtract the whole numbers and the fraction parts separately.
Step 3 - Subtract the whole numbers.
Step 4 - Subtract the fractions as usual. - Reduce the result to lowest terms, if necessary.
Step 5 - Combine the results from steps 3 and 4 to form your answer.

Example 1: $5\frac{5}{6} - 2\frac{1}{2} = (5-2) + (\frac{5}{6}-\frac{1}{2}) = 3 + (\frac{5}{6}-\frac{3}{6}) = 3 + \frac{2}{6} = 3 + \frac{1}{3} = 3\frac{1}{6}$

Example 2: $5\frac{1}{2} - 2\frac{2}{3} = 4\frac{3}{2} - 2\frac{2}{3} = (4-2) + (\frac{3}{2}-\frac{2}{3}) = 2 + (\frac{9}{6}-\frac{4}{6}) = 2\frac{5}{6}$

Note how $\frac{2}{3}$ is larger than $\frac{1}{2}$ (We can't subtract $\frac{2}{3}$ from $\frac{1}{2}$). We had to change $5\frac{1}{2}$ into $4\frac{3}{2}$.

Basically, we borrowed 1 from the 5 as follows: $5\frac{1}{2} = 4 + 1\frac{1}{2} = 4 + \frac{3}{2} = 4\frac{3}{2}$

Now you try it: Work the following problems on scratch paper (Use either method 1 or 2).

A) $2\frac{1}{2} + 3\frac{1}{4} = ?$ **B)** $4\frac{1}{8} + \frac{3}{4} = ?$ **C)** $5\frac{7}{8} + 7\frac{1}{2} = ?$ **D)** $2\frac{7}{8} + 5\frac{4}{5} + 1\frac{7}{10} = ?$

E) $3\frac{1}{2} - 2\frac{1}{4} = ?$ **F)** $4\frac{3}{4} - \frac{1}{8} = ?$ **G)** $7\frac{1}{2} - 5\frac{7}{8} = ?$ **H)** $5\frac{4}{5} + 1\frac{7}{10} - 2\frac{7}{8} = ?$

ANSWERS: A) 5 ¾ B) 4 ⅞ C) 13 ⅜ D) 10 ⅜ E) 1 ¼ F) 4 ⅝ G) 1 ⅝ H) 4 ⅝