

Math Help Sheet: Factoring

1st: Rearrange the terms if necessary.

Example: $8x^2 + 18 = 30x$ becomes $8x^2 - 30x + 18 = 0$

2nd: Factor out a monomial, if possible.

Example: $8x^2 - 30x + 18 = 0$ becomes $2(4x^2 - 15x + 9) = 0$

3rd: Consider known patterns:

$$a^2 - b^2 \rightarrow (a + b)(a - b)$$

$$a^2 + 2ab + b^2 \rightarrow (a + b)^2$$

$$a^2 - 2ab + b^2 \rightarrow (a - b)^2$$

$$a^3 + b^3 \rightarrow (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 \rightarrow (a - b)(a^2 + ab + b^2)$$

Examples:

$$49x^2 - 25 = (7x + 5)(7x - 5)$$

$$9x^2 + 12x + 4 \rightarrow \text{Try } (3x)^2 + 2(3x)(2) + (2)^2 = (3x + 2)^2$$

$$4x^2 - 15x + 9 \rightarrow \text{Try } (2x)^2 - 2(2x)(3) + 3^2 \text{ This is prime.}$$

$$27x^3 + 8 = (3x + 2)(9x^2 - 6x + 4)$$

$$x^3 - 125 = (x - 5)(x^2 + 5x + 25)$$

4th: For $x^2 + bx + c$, when the coefficient of x^2 is 1, we set up as follows:

With $b > 0$ and $c > 0 \rightarrow (x + \square)(x + \square)$ Example: $x^2 + 5x + 6 = (x + 2)(x + 3)$

With $b < 0$ and $c > 0 \rightarrow (x - \square)(x - \square)$ Example: $x^2 + 5x + 6 = (x - 2)(x - 3)$

With $c < 0 \rightarrow (x + \square)(x - \square)$ Example: $x^2 + 5x + 6 = (x + 2)(x - 3)$

5th: For $ax^2 + bx + c$, consider the factors of a and c . Use trial and error.

Example: $4x^2 - 15x + 9 = \text{what?}$ Try $(4x - \square)(x - \square)$ and $(2x - \square)(2x - \square)$

Try using 3 and 3, or 9 and 1 for the values of \square .

$(4x - 9)(x - 1)$ **No** $(2x - 9)(2x - 1)$ **No** $(4x - 1)(x - 9)$ **No** $(2x - 1)(2x - 9)$ **No**

$(4x - 3)(x - 3)$ **Yes** $(2x - 3)(2x - 3)$ **No**

6th: Use the zero-property to find solutions. Examples:

$$8x^2 - 30x + 18 = 0$$

$$2(4x - 3)(x - 3) = 0$$

$$4x - 3 = 0 \text{ or } x - 3 = 0$$

$$x = 3/4 \text{ or } x = 3$$

$$9x^2 - 18 = 27x$$

$$3(3x^2 - 7x - 6) = 0$$

$$3(3x + 2)(x - 3) = 0$$

$$x = -2/4 \text{ or } x = 3$$

Now you try it!

1. Factor $x^2 - 9$

2. Factor $4x^2 + 6x + 9$

3. Factor $3x^2 - 12x + 27$

4. Solve $x^2 - x - 6 = 0$

5. Solve $4x^2 - 3x = 1$

6. Solve $24x^2 - 4 = 4x$

1. $(x + 3)(x - 3)$ 2. $(2x + 3)^2$ 3. $3(x - 3)^2$ 4. $x = -2$ or $x = 3$ 5. $x = -1/4$ or $x = 1$ 6. $x = 1/2$ or $x = -1/3$