

Math Help Sheet: Finding the Least Common Denominator (L.C.D.)

Method 1: By inspection.

The L.C.D. is always the largest denominator multiplied by a whole number such as 1, 2, 3, ... Basically, we are trying successively larger multiples of the largest denominator (as a guess) and then seeing if the guess is evenly divided by the other denominator.

Example: $\frac{1}{6}$ and $\frac{3}{8}$ Try $8 \times 1 = 8$, does 8 divide evenly by 6? → NO
Try $8 \times 2 = 16$, does 16 divide evenly by 6? → NO

Try $8 \times 3 = 24$, does 24 divide evenly by 6? → YES, so **L.C.D. = 24**

Now you try it:

$$\frac{2}{3} \text{ and } \frac{4}{12} \text{ L.C.D.} = \underline{\quad}$$

$$\frac{5}{9} \text{ and } \frac{1}{6} \text{ L.C.D.} = \underline{\quad}$$

$$\frac{2}{7} \text{ and } \frac{3}{5} \text{ L.C.D.} = \underline{\quad}$$

Answers: 12 18 35

Method 2: Inspection with multiplication table assistance.

The multiplication table can always help when working with two fractions with denominators of 12 or less. Basically, we are looking at successively larger multiples of the largest denominator (as a guess) and then seeing if the guess is one of the multiples of the other denominator.

Example: $\frac{1}{6}$ and $\frac{3}{8}$

Find the lowest multiple of 8 that is also a multiple of 6.

Is this a multiple of 6? →

$$1 \times 8 = 8 \quad \text{NO}$$

$$1 \times 6 = 6$$

Is this a multiple of 6? →

$$2 \times 8 = 16 \quad \text{NO}$$

$$2 \times 6 = 12$$

Is this a multiple of 6? →

$$\boxed{3 \times 8 = 24} \text{ YES, so } \boxed{\text{L.C.D.} = 24}$$

$$3 \times 6 = 18$$

$$4 \times 8 = 32$$

$$\boxed{4 \times 6 = 24}$$

$$5 \times 8 = 40$$

$$5 \times 6 = 30$$

Now you try it using the multiplication tables (page 32):

$$\frac{2}{3} \text{ and } \frac{4}{12} \text{ L.C.D.} = \underline{\quad}$$

$$\frac{5}{9} \text{ and } \frac{1}{6} \text{ L.C.D.} = \underline{\quad}$$

$$\frac{2}{7} \text{ and } \frac{3}{5} \text{ L.C.D.} = \underline{\quad}$$

Answers: 12 18 35

We can often use the multiplication tables to handle several small denominators.

1 x 9 = 9
 2 x 9 = 18
 3 x 9 = 27
4 x 9 = 36
 5 x 9 = 45
 6 x 9 = 54
 7 x 9 = 63
 8 x 9 = 72
 9 x 9 = 81

1 x 6 = 6
 2 x 6 = 12
 3 x 6 = 18
 4 x 6 = 24
 5 x 6 = 30
6 x 6 = 36
 7 x 6 = 42
 8 x 6 = 48
 9 x 6 = 54

1 x 4 = 4
 2 x 4 = 8
 3 x 4 = 12
 4 x 4 = 16
 5 x 4 = 20
 6 x 4 = 24
 7 x 4 = 28
 8 x 4 = 32
9 x 4 = 36

Consider $\frac{1}{6}$ and $\frac{1}{4}$ and $\frac{5}{9}$

The L.C.D. is 36

Methods 1 and 2 are most useful with two denominators or three relatively small denominators.

Could you find the L.C.D. of $\frac{5}{14}$ and $\frac{9}{20}$ and $\frac{6}{35}$ with methods 1 or 2?

Method 3: A general method for finding the least common denominator.

Basically, we note how many times the denominators can be evenly divided by prime numbers. The divisors to try in order are 2, 3, 5, 7, 11, 13, 17, 19, 23, . . .

- Step 1: Write the denominators in a row (with proper spacing).
 Step 2: Draw a division line under the row of numbers like this:)_____.
 Step 3: Choose the smallest divisor from the above list so that at least one number divides evenly.
 Step 4: Write the divisor on the left.
 Step 5: For each number that divides evenly, writes the result below the line.
 If a number does not divide evenly, just copy that number below the line.
 Step 6: We now have a new row of numbers below. If they are not all 1's then go back to step 2.
 Step 7: Multiply all of the divisors used (the numbers on the left). This is the L.C.D.

Example: $\frac{1}{6}$ and $\frac{1}{2}$
 $2 \times 2 \times 2 \times 3 = 24$
 so L.C.D. = 24

2) 6 8
 2) 3 4
 2) 3 2
 3) 3 1
 1 1

Example: $\frac{1}{2}$ and $\frac{5}{2}$ and $\frac{5}{2}$
 $2 \times 2 \times 3 = 12$
 so L.C.D. = 12

2) 2 3 12
 2) 1 3 6
 3) 1 3 3
 1 1 1

Example: $\frac{5}{12}$ and $\frac{4}{21}$

$2 \times 2 \times 3 \times 7 = 84$
so L.C.D.= 84

$$\begin{array}{r} 2 \) \ 12 \ 21 \\ 2 \) \ \underline{6} \ 21 \\ 3 \) \ \underline{3} \ 21 \\ 7 \) \ \underline{1} \ 7 \\ 1 \ \ \ 1 \end{array}$$

Example: $\frac{16}{39}$ and $\frac{14}{63}$

$3 \times 3 \times 7 \times 13 = 819$
so L.C.D.= 819

$$\begin{array}{r} 3 \) \ 39 \ 63 \\ 3 \) \ \underline{13} \ 21 \\ 7 \) \ \underline{13} \ 7 \\ 13 \) \ \underline{13} \ 1 \\ 1 \ \ \ 1 \end{array}$$

Now you try it. Complete problems A, B, and C.

A) Find the L.C.D. of $\frac{1}{9}$ and $\frac{3}{8}$ and $\frac{5}{12}$

We set up the first row for you. \rightarrow) 9 8 12 (Place a 2 to the left in this line)
 Divide the above row by 2 (where possible) \rightarrow) _____ (Bring down the 9, a 4, and a 6)
 Divide the above row by 2 (where possible) \rightarrow) _____ (Bring down the 9, a 2, and a 3)
 Divide the above row by 2 (where possible) \rightarrow) _____ (Bring down the 9, a 1, and a 3)
 Divide the above row by 3 (where possible) \rightarrow) _____ (Bring down a 3, a 1, and a 1)
 Divide the above row by 3 (where possible) \rightarrow _____ (You should now have all 1's)
 Multiply 2, 2, 2, 3, and 3 to get the L.C.D. \rightarrow L.C.D. = _____

B) Consider $\frac{1}{6}$ and $\frac{14}{45}$) 6 45

L.C.D.= _____

C) Consider $\frac{5}{14}$ and $\frac{9}{20}$ and $\frac{6}{35}$) 14 20 35

L.C.D.= _____

Answers: A) 72 B) 90 C) 140