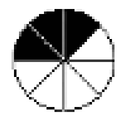
Math Help Sheet: Multiplying and Dividing Fractions

Parts of a fraction → $\frac{NUMERATOR}{DENOMINATOR}$ = NUMERATOR ÷ DENOMINATOR

In a fraction, the **denominator** (bottom number) tells us how many parts the whole is <u>divided into</u>, and the **numerator** (top number) tells us how many of those parts we're dealing with (the multiple).

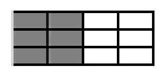
Consider a pizza cut into eight even pieces. Suppose you had three pieces.
You would have three parts out of eight, or 3 multiples of 1/8th of a pizza.



In terms of a fraction, you would have $\frac{3}{8}$ x 1 pizza = $\frac{3}{8}$ pizza.

Now consider a candy bar, which is easily divided into 12 pieces. →

Suppose you divided the bar in two, then took ½ of the bar.
You have 2 groups of 6 pieces, or 2 groups of ½ bar. This is ½ bar. →



Counting pieces: 12 pieces ÷ 2 = 6 pieces OR $\frac{1}{2}$ of 12 pieces = 6 pieces

Using fractions: 1 bar ÷ 2 = ½ bar OR $\frac{1}{2}$ of 1 bar = $\frac{1}{2}$ bar

NOTE: ½ of a (quantity) = (quantity) x $\frac{1}{2}$ = (quantity) ÷ $\frac{2}{1}$

The above example demonstrates an important point: We can divide by multiplying by the **reciprocal**.

When the product of two numbers is 1, one number is said to be the **reciprocal** of the other.
Since $\frac{2}{1}$ x $\frac{1}{2}$ = 1, we know that $\frac{1}{2}$ is the reciprocal of $\frac{2}{1}$, and that $\frac{2}{1}$ is the reciprocal of $\frac{1}{2}$. Other examples of values that are reciprocals of each other are as follows: $\frac{1}{6}$ and $\frac{6}{1}$, $\frac{2}{3}$ and $\frac{3}{2}$, $\frac{17}{3}$ and $\frac{3}{17}$.
Notice that for a fraction, its **reciprocal** is obtained by "flipping the fraction over".
Also, note that when a whole number is involved we turn it into a fraction with a denominator of 1.

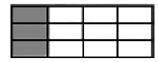
When working word problems, sometimes we need do fraction multiplication or division.

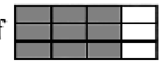
- To find a fraction of a quantity, we must multiply by the fraction. (Note the word "of")
  Example: How many pieces in three fourths of one bar?
  $\frac{3}{4}$ x 12 pieces = 9 pieces (As shown on the next page).

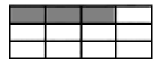
- When we are dividing up a quantity, we may need to divide by a fraction. (We multiply by the reciprocal.)
  Example: How many two-thirds bar servings can you provide with 6 candy bars?
  $6 \div \frac{2}{3} = \frac{6}{1} \times \frac{3}{2} = 9$ servings (As shown on the next page).

Now we can address the fine points of multiplying and dividing fractions.

Let's start with a pictorial approach to finding two thirds of three fourths of one 12-piece candy bar.

Three fourths of one bar (or $\frac{3}{4}$ bar) is 3 multiples of $\frac{1}{4}$ bar. 3 x  = 

Two thirds of ($\frac{3}{4}$) bar can be considered to be 2 multiples of $\frac{1}{3}$ of 

2 x  =  = ANSWER

The above exercise clearly shows that two-thirds of three-fourths of one bar is 6 pieces, or ½ bar.

In many ways, multiplying two fractions is easier than adding them. When multiplying two fractions we do not need to find a common denominator as we would need to do in order to add them. All we need to do is to multiply the numerators together (top numbers), and multiply the denominators together (bottom numbers). Now lets use fraction multiplication to work the above problem.

Two-thirds of three-fourths of a candy bar = $\frac{2}{3}$ x $\frac{3}{4}$ bar = $\frac{2\times3}{3\times4}$ bar = $\frac{6}{12}$ bar, which is $\frac{1}{2}$ bar.

Note: Our answer needed to be simplified by expressing it as a smaller equivalent fraction.

Equivalent fractions are fractions that represent the same value such as $\frac{2}{3}, \frac{4}{6}$, and $\frac{6}{9}$

Note: The value of a fraction is not changed if both its numerator and denominator are multiplied by the same number, or divided by the same number. Example: $\frac{1}{2}=\frac{1\times6}{2\times6}=\frac{6}{12}$ (We multiplied numerator and denominator by 6) $\frac{6}{12}=\frac{6\div6}{12\div6}=\frac{1}{2}$ (We divided numerator and denominator by 6)

When multiplying fractions we may use canceling, which makes it easier to get a reduced result. When the numerator of one fraction and the denominator of the other fraction have a common multiple then we immediately divide both values by that multiple.

Consider the problem above: $\frac{2}{3}$ x $\frac{3}{4}$ = $\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}$ x $\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}}$ = $\frac{1}{2}$ (Note how the old values are replaced)

More examples: $\frac{5}{6}$ x $\frac{3}{4}$ = $\frac{5}{\underset{2}{\cancel{6}}}$ x $\frac{\overset{1}{\cancel{3}}}{4}$ = $\frac{5}{8}$; $\frac{2}{3}$ x $\frac{9}{8}$ = $\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}$ x $\frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{8}}}$ = $\frac{3}{4}$; 6 ÷ $\frac{2}{3}$ = $\frac{\overset{3}{\cancel{6}}}{1}$ x $\frac{3}{\underset{1}{\cancel{2}}}$ = 9

$\frac{5}{6}$ x 9 = $\frac{5}{\underset{2}{\cancel{6}}}$ x $\frac{\overset{3}{\cancel{9}}}{1}$ = $\frac{15}{2}$ = $\frac{14+1}{2}$ = $7\frac{1}{2}$ ← We convert an improper fraction into a mixed number.

Now You Try It:

A) $\frac{9}{16}$ x $\frac{2}{3}$ = ? B) $\frac{9}{16}$ ÷ $\frac{2}{3}$ = ? C) $\frac{9}{16}$ ÷ 6 = ? D) 4 x $\frac{3}{8}$ = ? E) $\frac{3}{4}$ ÷ $\frac{2}{3}$ = ?

Answers: A) 3/8 B) 27/32 C) 3/32 D) 3/2 or 1 ½ E) 9/8 or 1 1/8