Math Help Sheet
Solving Quadratic Equations by Completing the Square

Given an expression $x^2 + bx$, if we add a constant term (c) equal to the square of $\frac{1}{2}$ the coefficient of x, $\left(\frac{b}{2}\right)^2$, to obtain a square trinomial.

General Form: $x^2 + bx + c$

$x^2 + bx + \left(\frac{1}{2}b\right)^2$

$\left(x + \frac{1}{2}b\right)^2$

Example:		
	$x^2 + 6x + \_\_\_$	Find the value that forms a square trinomial
	$x^2 + 6x + (3)^2$	$\left(\frac{1}{2}b\right)^2 = \left(\frac{6}{2}\right)^2 = (3)^2$
	$x^2 + 6x + 9$	Here, $c = 9$
	$(x + 3)^2$	Write as a binomial

Fill in the blank for each expression below with the value of c that makes the trinomial a perfect square. Then rewrite each trinomial as a square of a binomial.

1) $y^2 + 4y + \_\_\_$.

$= (\_\_\_\_\_)^2$

2) $y^2 - 2y + \_\_\_$.

$= (\_\_\_\_\_)^2$

3) $z^2 - 3z + \_\_\_$.

$= (\_\_\_\_\_)^2$

4) $v^2 - \frac{v}{5} + \_\_\_$.

$= (\_\_\_\_\_)^2$

5) $w^2 + \frac{4}{3}w + \_\_\_$.

$= (\_\_\_\_\_)^2$

Answers:
1) $4, (y+2)^2$; 2) $1, (y-1)^2$; 3) $\frac{9}{4}, \left(z - \frac{3}{2}\right)^2$;
4) $\frac{1}{100}, \left(v - \frac{1}{10}\right)^2$; 5) $\frac{4}{9}, \left(w + \frac{2}{3}\right)^2$

We can complete the square to solve quadratic equations.

$\boldsymbol{x^2 + 3x = 4}$

$x^2 + 3x + \left(\frac{3}{2}\right)^2 = 4 + \left(\frac{3}{2}\right)^2$

Complete the square on the left side.
(be sure to add the same value to the right side)

$x^2 + 3x + \left(\frac{3}{2}\right)^2 = 4 + \frac{9}{4}$

$\left(x + \frac{3}{2}\right)^2 = \frac{25}{4}$

Write the left side as the square of a binomial.

$\sqrt{\left(x + \frac{3}{2}\right)^2} = \pm\sqrt{\frac{25}{4}}$

Take the square root of both sides.

$x + \frac{3}{2} = \pm\frac{5}{2}$

Simplify the right side.

$x = -\frac{3}{2} \pm \frac{5}{2}$

Solve for x.

$x = \{1, -4\}$

Simplify the values for x.
(Remember, quadratic equations have 2 roots, even though they may be equal.)

$\boldsymbol{2x^2 + 5x + 2 = 0}$

$2x^2 + 5x = -2.$

Subtract 2 from each side.

$x^2 + \frac{5}{2}x = -1$

Divide each side by 2 so that the coefficient of x^2 is 1.

$x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = -1 + \left(\frac{5}{4}\right)^2$

Complete the square on the left side,
adding the same value to the right side also.

$x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = -1 + \frac{25}{16}$

Simplify.

$\left(x + \frac{5}{4}\right)^2 = \frac{9}{16}$

Write the left side as the square of a binominal and simplify the right side.

$\sqrt{\left(x + \frac{5}{4}\right)^2} = \pm\sqrt{\frac{9}{16}}$

Take the square root of both sides of the equation.

$x + \frac{5}{4} = \pm\frac{3}{4}$

Simplify the right side.

$x = -\frac{5}{4} \pm \frac{3}{4}$

Solve for x.

$x = \left\{-2, -\frac{1}{2}\right\}$

Simplify the values of x.

Solve the following by completing the square.

$$y^2 - 4y + 3 = 0$$

Answer:

$y^2 - 4y + 3 = 0$

$y^2 - 4y = -3$

$y^2 - 4y + 4 = -3 + 4$

$(y-2)^2 = 1$

$\sqrt{(y-2)^2} = \pm\sqrt{1}$

$y - 2 = \pm 1$

$y = 2 \pm 1$

$y = \{3, 1\}$

$$3w^2 + 6w = 12$$

Answer:

$\frac{3w^2+6w}{3} = \frac{12}{3}$

$w^2 + 2w = 4$

$w^2 + 2w + (1)^2 = 4 + (1)^2$

$w^2 + 2w + 1 = 5$

$(w+1)^2 = 5$

$\sqrt{(w+1)^2} = \pm\sqrt{5}$

$w + 1 = \pm\sqrt{5}$

$w = -1 \pm \sqrt{5}$