

Math Help Sheet:
Solving Quadratic Equations with Square Roots and Factoring

There are several ways to solve quadratic equations. Here, we will briefly review how to solve quadratic equations with square roots and factoring, giving an example of each.

Solving with square roots:

When a squared quantity is equal to a value, solve by taking the square root of both sides.

$$x^2 = 7;$$

$$\sqrt{x^2} = \pm\sqrt{7}$$

$$x = \pm\sqrt{7}.$$

$$(u - 5)^2 = 16$$

$$\sqrt{(u - 5)^2} = \pm\sqrt{16}$$

$$u - 5 = \pm 4$$

$$u = 5 \pm 4$$

$$u = 9 \text{ or } u = 1$$

Practice: $x^2 = 17$

Answer:
 $x^2 = 17$
 $x = \pm\sqrt{17}$

Practice: $(w + 2)^2 = 9$

Answer:
 $(w + 2)^2 = 9$
 $\sqrt{(w + 2)^2} = \pm\sqrt{9}$
 $w + 2 = \pm 3$
 $w = -2 \pm 3$
 $w = 1 \text{ or } w = -5$

Solving by Factoring:

If the product is zero, at least one of the factors must be zero. Find the factors and set each factor equal to zero to solve.

$$x^2 + 8x + 12 = 0;$$

$$(x + 6)(x + 2) = 0;$$

$$x + 6 = 0 \text{ or } x + 2 = 0;$$

$$x = -6 \text{ or } x = -2.$$

Practice: $x^2 = 7x - 12$

Answer:

$$\begin{aligned}x^2 &= 7x - 12 \\x^2 - 7x + 12 &= 0 \\(x - 3)(x - 4) &= 0 \\x - 3 = 0 \text{ or } x - 4 &= 0 \\x &= 3 \text{ or } x = 4\end{aligned}$$

These two methods may be combined to solve a quadratic equation. Below, start by factoring, then find the square root of each side to continue solving.

Practice: $x^2 + 2x + 1 = 8$

Answer:

$$\begin{aligned}x^2 + 2x + 1 &= 8 \\(x + 1)^2 &= 8 \\\sqrt{(x + 1)^2} &= \pm\sqrt{8} \\x + 1 &= \pm\sqrt{8} = \pm\sqrt{4}\sqrt{2} = \pm 2\sqrt{2} \\x &= -1 \pm 2\sqrt{2}\end{aligned}$$