Math Help Sheet:

Solving Quadratic Equations with the Quadratic Formula

The quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

It can be used to solve for the root(s) of any quadratic equation in standard form of $ax^2 + bx + c = 0$. If a quadratic equation is not equal to zero, we must first set the quadratic equation equal to zero using the rules of algebra.

In other words, x depends only on the numerical coefficients for "a", "b" and "c" of the original equation in standard form equal to zero. To solve for the solution(s) we substitute the numeric values of the "a", "b" and "c" coefficients into the quadratic formula.

A NOTE OF CAUTION: With the equation in standard form equal to zero, the sign always goes with the coefficient to the right of the sign. For example if the equation is in the form of $ax^2 - bx + c = 0$, it is the same as $ax^2 + (-b)x + c = 0$. So in this case the b coefficient is a negative.

Let's look at a few examples.

Example 1: $2x^2 + 9x + 1 = -3$.

We must first set this quadratic equation equal to zero by adding 3 to both sides

$$x^2 + 9x + 1 + 3 = -3 + 3$$

$$2x^2 + 9x + 4 = 0$$

For this problem, $a = 2$, $b = 9$ and $c = 4$. Substituting these terms into the quadratic formula we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-9 \pm \sqrt{9^2 - 4(2)(4)}}{2(2)} = \frac{-9 \pm \sqrt{81 - 32}}{4} = \frac{-9 \pm \sqrt{49}}{4} = \frac{-9 \pm 7}{4}$$

$$x = \frac{-9 + 7}{4} = \frac{-2}{4} = -\frac{1}{2} \quad OR \quad x = \frac{-9 - 7}{4} = \frac{-16}{4} = -4$$

Example 2: $2x^2 - 9x + 4 = 0$.

For this problem, $a = 2$, $b = -9$ and $c = 4$. Substituting these terms into the quadratic formula we have

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-9) \pm \sqrt{(-9)^2-4(2)(4)}}{2(2)} = \frac{9 \pm \sqrt{81-32}}{4} = \frac{9 \pm \sqrt{49}}{4} =$$

$$x = \frac{9+7}{4} = \frac{16}{4} = 4 \qquad OR \qquad x = \frac{9-7}{4} = \frac{2}{4} = \frac{1}{2}$$

So as you can see by the "b" coefficient being negative instead of positive for the "same" quadratic equation changes the two solutions in Example 1 from being negative to being positive for example 2. This is why it is important to pay attention to all signs when solving a quadratic equation

Example 3: $3x^2 + 5x + 6 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} == \frac{-5 \pm \sqrt{5^2 - 4(3)(6)}}{2(3)} = \frac{-5 \pm \sqrt{25 - 72}}{6}$$

$$x = \frac{-5 + \sqrt{-67}}{6}$$

Under the radical symbol above we have –67. Since we cannot take a square root of a negative number, this problem has no solution.

Below are four practice problems to test your knowledge using the quadratic formula and then check your answers with the solutions on the next two pages.

Remember, the roots are $= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Problem 1: $y^2 - 4y + 3 = 0$

Problem 2: $4w^2 - 2w = 5$

Problem 3: $x^2 = 2x + 1$

Problem 4: $5m^2 - 17 = 0$

Problem 1 Solution: $y^2 - 4y + 3 = 0$

$a = 1 \quad b = -4 \quad c = 3$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

$$y = \frac{4 \pm \sqrt{16 - 12}}{2} = \frac{4 \pm \sqrt{4}}{2},$$

$$y = \frac{4+2}{2} = \frac{6}{2} = 3 \quad OR \quad y = \frac{4-2}{2} = \frac{2}{2} = 1$$

Problem 2 Solution: $4w^2 - 2w = 5$

First need to set the equation equal to zero by subtracting 5 from both sides so the equation becomes

$a = 4 \quad b = -2. \quad c$

$4w^2 - 2w - 5 = 5 - 5$

$4w^2 - 2w - 5 = 0$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(-5)}}{2(4)} = \frac{2 \pm \sqrt{4 + 80}}{8}$$

$$w = \frac{2 \pm \sqrt{84}}{8} = \frac{2 \pm \sqrt{(4)(21)}}{8} = \frac{2 \pm 2\sqrt{21}}{8} = \frac{1 \pm \sqrt{21}}{4}$$

$$w = \frac{1+\sqrt{21}}{4} \quad OR \quad w = \frac{1-\sqrt{21}}{4}$$

Problem 3 Solution: $x^2 = 2x + 1$

First need to set the equation equal to zero by subtracting 2x and1 from both sides so the equation becomes:

$x^2 - 2x - 1 = 2x - 2x + 1 \quad - 1$

$x^2 - 2x - 1 = 0$

$a = 1. b = -2, c = -1$

$$\frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2}$$

$$x = \frac{2 \pm \sqrt{(4)(2)}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

$$x = 1 + \sqrt{2} \quad OR \quad x = 1 - \sqrt{2}$$

Problem 4 Solution: $5m^2 - 17 = 0$

$a = 5, b = 0, c = -17$

Since there is not middle term for this equation the "b" coefficient is equal to zero.0^2

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0 \pm \sqrt{0^2 - 4(5)(-17)}}{2(5)} = \frac{\pm\sqrt{340)}}{10}$$

$$m = \frac{\pm\sqrt{(4)(85)}}{10} = \frac{\pm 2\sqrt{85}}{10} = \pm \frac{\sqrt{85}}{5}$$

$$m = \frac{+\sqrt{85}}{5} \quad OR \quad \frac{-\sqrt{85}}{5}$$