



Math Help Sheet: Solving Systems of Linear Equations

You have already done some work in graphing linear equations. What would happen if you graphed two of these equations on the same axes? You would get at least one line, and in most cases you would get two lines. If both equations are actually identical, their graphs would be the same straight line. In this case we say the equations are dependent.

Suppose instead that we get two different lines. Either they would be parallel lines or they would intersect in one point. If the equations graph into parallel lines, we say they are inconsistent equations. Equations which intersect in one point we call independent.

Many times you are given certain conditions pertaining to two unknowns and you are asked to find the unknowns. This involves solving a pair of simultaneous equations. One method of doing this is by graphing.

Suppose the two equations were $x + 2y = 3$ and $3x - y = 1$. The values of x and y would be the solution are those which satisfy (make true) both equations. The graph of an equation is the representation of all possible values which satisfy the equation. Therefore, the solution of these two equations would be any points which are on the graphs of both of these equations or the intersection of the lines. Consequently, if the lines were parallel, there would be no solutions. If the lines were the same, there would be many solutions.

Let's look at an example.

Suppose you wish to know how old your cousins Bill and Betty are. Your aunt tells you that the sum of their ages is 30 years and Bill is 2 years older than Betty. How old are they?

Let x = Bill's age and y = Betty's age.

Then $x + y = 30$ and $x - y = 2$.

You can solve this problem by first graphing $x + y = 30$ and then $x - y = 2$ on the same axes. The point where the two lines meet is the answer.

$$x + y = 30 \text{ or } y = 30 - x$$

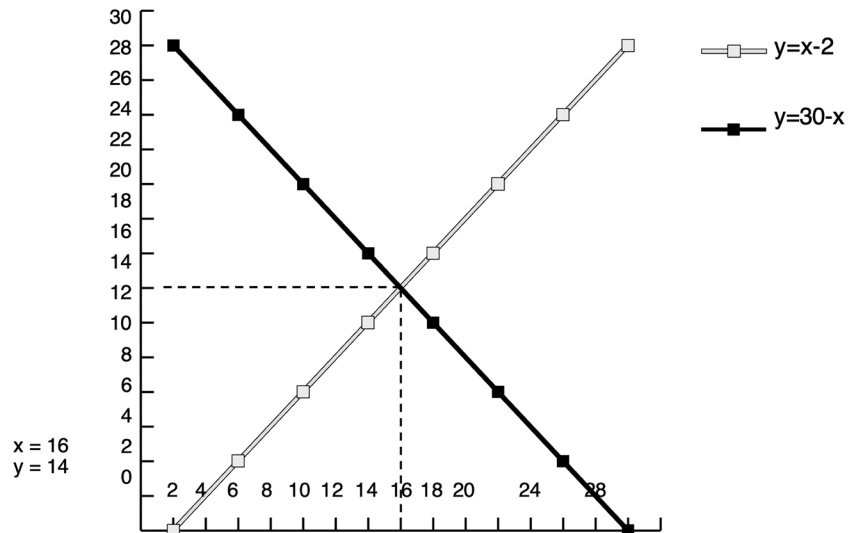
Plot a few points to find our line.

x	y
2	28
6	24
10	20

$$x - y = 2 \text{ or } y = x - 2$$

Plot a few points to find our line.

x	y
2	0
6	4
10	8



Let's look at several other examples.

A. $y = x + 5$
 $y = x + 6$

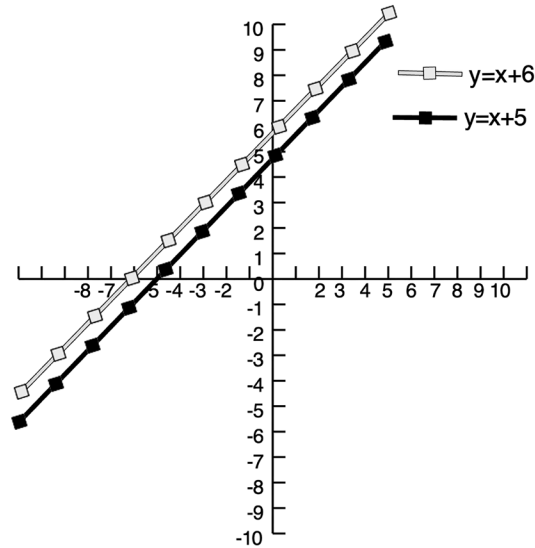
Graph each of these equations on the same axes.

$$y = x + 5$$

$$y = x + 6$$

x	y
0	5
1	6
2	7

x	y
0	6
1	7
2	8



These lines are parallel so there are no solutions. Could you have known this without actually graphing the equation? Yes.

It seems logical that no number plus 5 could give you the same answer as if you had added 6 to the same number. Since $y = x + 5$ and $y = x + 6$, $x + 5 = x + 6$, and this isn't reasonable. If you add 6 to a number, you won't get the same result as if you add 5 to it.

B. $y = x - 1$
 $3y = 3x - 3$

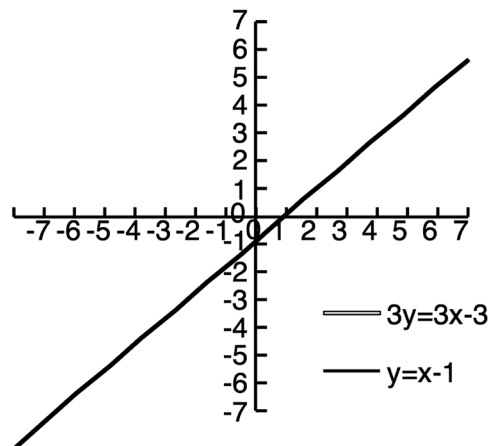
Graph both equations on the same axes.

$y = x - 1$

$3y = 3x - 3$

x	y
0	-1
1	0
2	1

x	y
0	-1
1	0
2	1



These two lines are identical so there is an infinite number of solutions or points which satisfy both equations. Again we can tell this without graphing the equations first.

The second equation is $3y = 3x - 3$.

Divide both sides by 3: $y = x - 1$.

This is the same as the first equation.

C. $y = x - 1$
 $y = 3 - x$

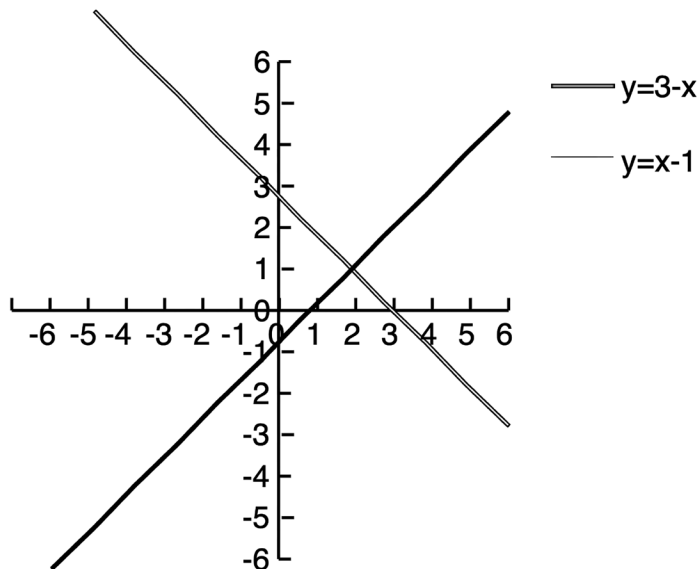
Graph both equations on the same axes.

$y = x - 1$

$y = 3$

x	y
0	-1
1	0
2	1

x	y
0	3
1	2
2	1



The lines intersect at the point (2, 1) so this is the solution. This point satisfies both equations.

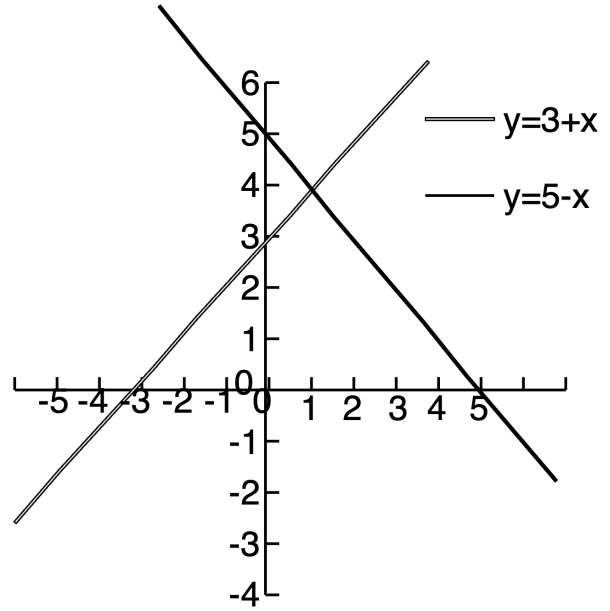
D. $y = 5 - x$
 $y = 3 + x$

$y = 5 - x$

$y = 3 + x$

$$\begin{array}{r|l} x & y \\ \hline 0 & 5 \\ 1 & 4 \\ 2 & 3 \end{array}$$

$$\begin{array}{r|l} x & y \\ \hline 0 & 3 \\ 1 & 4 \\ 2 & 5 \end{array}$$



The solution is (1, 4).

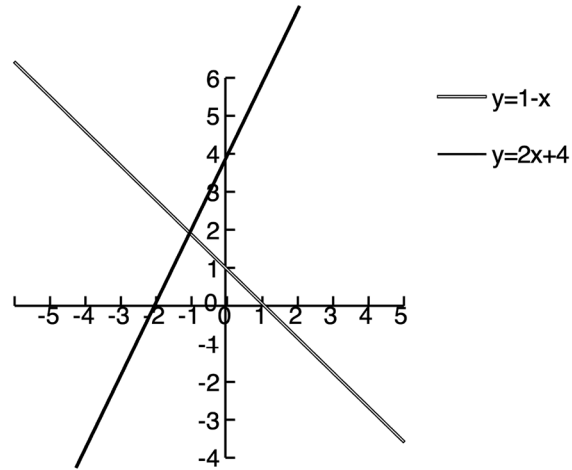
E. $y = 2x + 4$
 $x + y = 1$

$y = 2x + 4$

$x + y$
or $y = 1 - x$

$$\begin{array}{r|l} x & y \\ \hline 0 & 4 \\ 1 & 6 \\ 2 & 8 \end{array}$$

$$\begin{array}{r|l} x & y \\ \hline 0 & 1 \\ 1 & 0 \\ 2 & -1 \end{array}$$



The solution appears to be (-1, 2).
Let's see if that is correct.

$y = 2x + 4$

$x + y = 1$

$2 = 2(-1) + 4 ?$

$(-1) + 2 = 1$ YES

$2 = (-2) + 4$ YES

$$\begin{aligned} \text{F. } \quad x + y &= 5 \\ x - y &= 3 \end{aligned}$$

$$\begin{aligned} x + y &= 5 \\ \text{or } y &= 5 - x \end{aligned}$$

$$\begin{array}{r|l} x & y \\ \hline 0 & 5 \\ 1 & 4 \\ 2 & 3 \end{array}$$

$$\begin{aligned} x - y &= 3 \\ \text{or } y &= x - 3 \end{aligned}$$

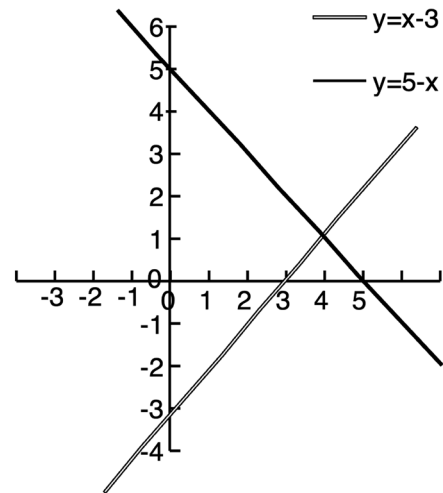
$$\begin{array}{r|l} x & y \\ \hline 0 & -3 \\ 1 & -2 \\ 2 & -1 \end{array}$$

The solution is (4, 1).

Let's check that!

$$\begin{array}{r} x + y = 5 \\ \quad ? \\ 4 + 1 = 5 \\ \quad ? \\ 5 = 5 \end{array}$$

$$\begin{array}{r} x - y = 3 \\ \quad ? \\ 4 - 1 = 3 \\ \quad ? \\ 3 = 3 \end{array}$$



Now you try to solve the following by graphing:

$$\begin{aligned} x + 2y &= 5 \\ x &= 3y \end{aligned}$$

Did you get (3, 1) as the solution? If you did, you are correct.