Math Help Sheet: Solving Word Problems Algebraically

When solving word problems, failure to show a complete algebraic solution will result in loss of points. There are five important steps needed to properly work through a word problem:
1) Identify the values you are working with. 2) Choose variables and expressions for each value. 3) Set up equation(s) to model the problem. 4) Solve for the variable(s) and clearly state an answer for each of the values in the problem. 5) Check your answer against the words in the problem. Note: It is not enough to just make sure your equation(s) work. Perhaps you made an error in setup.

Let's look at an example word problem.
A jar of nickels, dimes, and quarters contains 200 silver coins totaling $23.50.
There are 4 more dimes than nickels. How many of each coin do you have?

One possible solution is below with the five steps modeled for you.

Consider what the problem is asking you to find and what you know that is relevant.

(nickels) + (dimes) + (quarters) = 200 coins
You have 4 more dimes than nickels.
Total amount = $23.50

Let d = number of dimes
Let (d – 4) = number of nickels
Let 200 – d – (d – 4) = number of quarters

Choose variable(s) to represent your unknown values.

Write equation(s) using the variable(s) you have chosen.
(A quarter is .25 of a dollar; a nickel is .05, and a dime is .10)

$.10d + .05(d - 4) + .25(200 - d - (d - 4)) = 23.50$

$.10d + .05d - .20 + .25(200 - d - (d - 4)) = 23.50$
$.15d + .25(204 - 2d) = 23.70$
$-.35d + 51 = 23.70$
$-.35d + 51 = -27.30$
$d = 78$

Solve your equation(s).
(Your approach may vary, but must be mathematically correct and well documented.)

d = 78 dimes
d – 4 = 74 nickels
200 – 78 – 74 = 48 quarters

Once you have values for your variable(s), check that you have answered the question(s) asked

Check your values using the words of the problem.

There are 200 coins: 78 + 74 + 48 = 200 √
The total value is $23.50:
78(.1) + 74 (.05) + 48(.25) = 7.8 + 3.7 + 12 = $23.50 √

This is another solution perspective for the same word problem. Either solution is an acceptable algebraic solution. On exams, variable selection is up to you. Full credit will be awarded to any correct algebraic solution that is completely documented.
Here is the word problem again:
A jar of nickels, dimes, and quarters contains 200 silver coins totaling $23.50.
There are 4 more dimes than nickels. How many of each coin do you have?

Consider what the problem is asking you to find and what you know that is relevant.

(nickels) + (dimes) + (quarters) = 200 coins
You have 4 more dimes than nickels.
Total amount = $23.50

Choose variable(s) to represent your unknown values.

Let q = number of quarters
Let n = number of nickels
Since we are limited to two variables, dimes must be expressed in q or n.
Let (n + 4) = number of dimes

Consider that you must have at least one unique equation per unique variable.
(In this case, we have two equations, so we may have two variables, but for problems with one equation, you will be limited to one variable).

Write equation(s) using the variable(s) you have chosen.
(A quarter is .25 of a dollar; a nickel is .05, and a dime is .10)

Equation A
$q + n + (n + 4) = 200$

Equation B
$.25q + .05n + .1(n + 4) = \$23.50$

rearrange A:
$q + 2n + 4 = 200$
$q = 196 – 2n$ ←(substitute this into B)
use substitution to solve B:
$.25(196 – 2n) + .1(n + 4) + .05n = 23.50$
$49 – .5n + .1n + 4 + .05n = 23.50$
$–.5n + .1n + 4 + .05n = 23.50 – 49 – 4$
$–.35n = –25.9$
$n = 75$ nickels
$n + 4 = 78$ dimes
$200 – 74 – 78 = 48$ quarters

Solve your equations.
(Your approach may vary, but must be mathematically correct and well documented.)

Once you have values for your variable(s), check that you have answered the question(s) asked

There are 200 coins: $78 + 74 + 48 = 200$ √
The total value is $23.50:
$78(.1) + 74(.05) + 48(.25) = 7.8 + 3.7 + 12$
$= \$23.50$ √

Check your values using the words of the problem.

Although the variable choices differed slightly in the two solutions shown, both followed the same five steps and led to the correct answer.