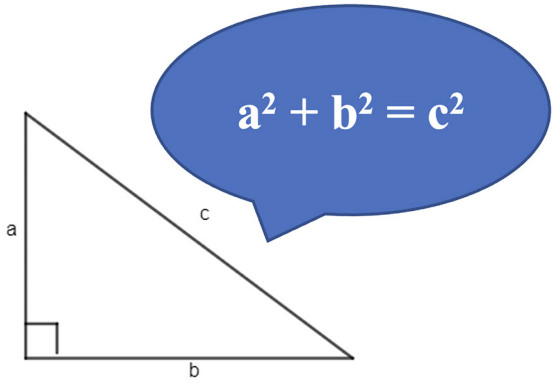
Math Help Sheet: The Pythagorean Theorem

The Pythagorean theorem is a very powerful geometric principle that governs the characteristics of all right triangles. The Pythagorean theorem states:

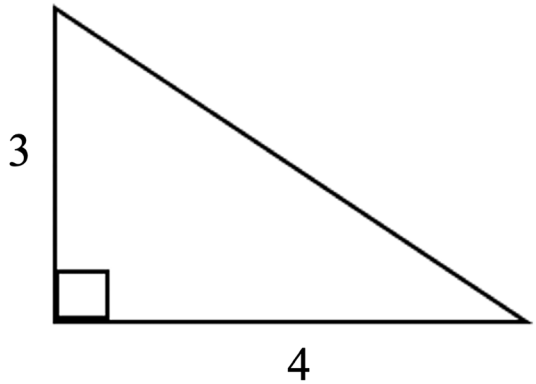
> "**For any right triangle, the square of length of the hypotenuse equals the sum of the square of each of the lengths of the legs.**"

Often people remember the formula "$\mathbf{a^2 + b^2 = c^2}$", where **c** is the length of the hypotenuse, and **a** and **b** are the lengths of the legs. The figure below shows that the right angle is marked with a small box. The hypotenuse is opposite the right angle and will always be the longest side.



This relationship holds for **ALL RIGHT TRIANGLES**. If this relationship does not hold, the triangle is not a right triangle. The Pythagorean Theorem also allows the easy calculation of any side of a right triangle if the lengths of the other 2 sides are known.
To solve for an unknown side of a right triangle: substitute the known lengths into the formula, and solve for your variable.

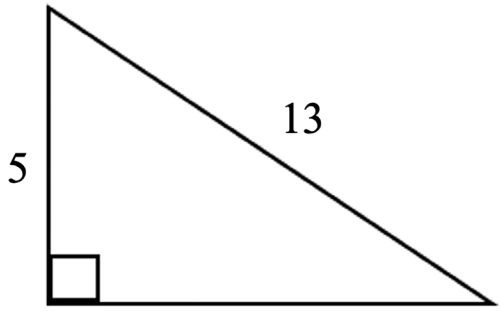
Find the length of the hypotenuse of the following right triangle:



c is missing, so substitute, a = 3 and b = 4
$3^2 + 4^2 = c^2$
$9 + 16 = 25 = c^2 = 5^2 \rightarrow c = 5$
The hypotenuse is 5 units long.

Some times the legs are not whole numbers. Given legs of 12.5 and 30, we have
$(12.5)^2 + (30)^2 = c^2$
$156.25 + 900 = 1{,}056.25$
$c = \sqrt{1056.25} = 32.5$

Find the length of the missing leg of the following right triangle:



b is missing, so substitute, a = 5 and c = 13
$5^2 + b^2 = 13^2$
$25 + b^2 = 169$ so $b^2 = 144 \rightarrow c = 12$
The bottom leg is 12 units long.

The Pythagorean theorem can also be used to prove whether or not a given set of lengths form a right triangle. To determine if the given lengths form a right triangle, simply substitute them into the equation, and if the equality holds, the lengths form a right triangle. However, if the equality does not hold, the lengths cannot form a right triangle.

Consider the following examples:

If a triangle has lengths 4, 5, and 6, is it a right triangle?
Substitute the lengths into the formula (remember the hypotenuse is always the longest side), a = 4, b = 5, c = 6.
We check to see if $4^2 + 5^2 = 6^2$
$4^2 + 5^2 = 16 + 25 = 41$, and $6^2 = 36$, so equality does not hold.
NO. The side lengths do not form a right triangle.

If a triangle has lengths 9, 15, and 12, is it a right triangle?
Substitute the lengths into the formula (remember the hypotenuse is always the longest side), a = 9, b = 12, c = 15.
We check to see if $9^2 + 12^2 = 15^2$
$9^2 + 12^2 = 81 + 144 = 225$, and $15^2 = 225$, so equality holds true.
YES. The side lengths form a right triangle.

Now you try it.

1. Find the hypotenuse of a right triangle with legs 15 and 36 units in length.

2. Find the missing leg of a right triangle with hypotenuse 15 and a leg 9 units in length.

3. Is a triangle with side lengths 12.5, 32.5, and 30 a right triangle?

a) 39 b) 12 c) yes